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## NET PRESENT VALUE AND REAL OPTIONS

by

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## Abstract

Real investment projects often come with some degree of potential flexibility which may result in changes in the original cashflow pattern. The question now arises whether those changes necessarily have to be evaluated using modern option methodology or whether the established NPV-framework can still be used for a correct evaluation.

## NET PRESENT VALUE AND REAL OPTIONS

**R. Vandenborre**

## 1. Introduction

In articles and books dealing with the evaluation of cash flows it is generally claimed that traditional capital budgeting methods or discounted cash flow approaches cannot cope with the "operating flexibility options and other strategic aspects" of various projects but that the application of option techniques results in the correct solution[1]. Or to quote another author : "The NPV rule is easy, but it makes the false assumption that the investment is either reversible or that it cannot be delayed"[2].

It is the purpose of this paper to add to this discussion, to clarify concepts and to try to evaluate the merits (or lack of them) of discounted cash flow methods.

## 2. The problem. A single period

We start the discussion with the example reported in the book referred to in the first footnote. The data of the problem can be summarized as follows:

- investment at start period 1: 104 million
- end of period payoffs: 180 with p = .5
  60 with p = .5
- riskfree rate = 8 %
- the project's specific cost of capital (obtained through the market valuation of a security with exactly the same payoff pattern) is 20%.
  The present value of the project using the information provided is 100 million. As the cost is 104 million, the project presents a negative net present value.

The author poses the question what the value of the project would be if the firm in question would possess a one-year license granting it the exclusive right to defer undertaking the project for a year. Investment cost a year from now would have grown to

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1 See e.g. L. Trigeorgis "Real Options", MIT Press, 1996.

2 A.K. Dixit and R.S. Pindyck "The options approach to capital investment", Harvard Business Review, May-June 1995.
For a good understanding, we would state that if a particular method is used inappropriately, the user is at fault, not the method.

(104).(1.08) = 112.32 million. For simplicity, the author maintains payoffs of respectively180 and 60 at end of period one[3].

Of course, if an undesirable situation in the course of the year develops, the firm will not carry out the project because revenue < cost (60 < 112.32). In the other case the project would become reality. Evaluating the option (start of period), we first solve for the valuation constants :

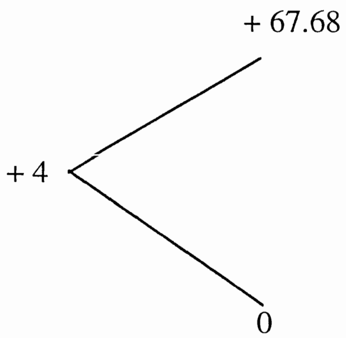
$$\alpha\,(180) + \beta\,(108) = (180 - 112.32) = 67.68$$

$$\alpha\,(60) + \beta\,(108) = 0$$

Consequently $\alpha = 0.564$ and $\beta = -0.313$. And the value of these flows is (0.564 x 100) + (- 0.313 x 100) or 25.10[4].

This value is not the value of the option because the flows on which this result is based do not constitute the complete option flows. And indeed, the author of the example calls this value : "the total value of the investment opportunity (expanded NPV) that incorporates the value of the option to defer"[5]. Because the 'incorporation' is not explicit and therefore somewhat unclear, it might be advisable to calculate the value of the option directly rather than in an ad hoc fashion (by using the addivity rule according to which the total value of a project ("expanded NPV") equals the original value plus the value of the option). Using the numbers of this example :

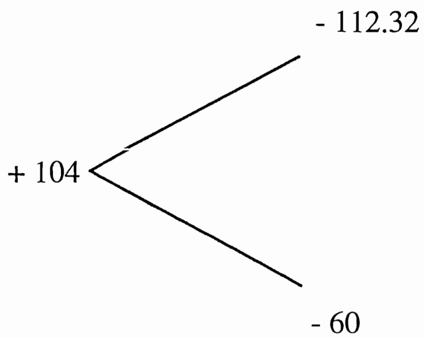
<u>basis : net results</u>




value option 29.10

or

<u>basis : changes in cash flows</u>




value option 29.10

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3 This assumption simplifies the situation considerably since the question of the size of a cost of capital for period two does not now arise.

4 The payoff combination (180 and 60) has market value 100 for a cost of capital of 20 %. The riskfree end of period value of 108 has of course a present value of 100 (rate 8 %).

5 L. Trigeorgis, ibid., p. 160. It would therefore be more direct to say that the result (25.1) is the outcome of an option-method valuation of the flows (67.68, 0).

We prefer the changes in cash flows approach.

The author remarks that valuing the flows (67.68, 0) at a cost of capital of 20 % would have resulted in an erroneous value of 28.2 instead of 25.1. And the conclusion then quickly follows that the NPV-method cannot be used in those situations[6].

Of course, as the option has potentially changed the flow pattern of the project, so has it changed its risk profile and therefore adjustment for risk should be adapted accordingly[7]. But it does therefore not follow that the NPV-methodology has become inappropriate. The NPV-methodology is, if correctly applied, sound ; moreover it can be applied to option flows. The total (expanded) flows can be broken up in the original cash flows and the potential changes to these because of 'option' possibilities. Rejecting the usefulness of the NPV-methodology boils down to pretending that the appropriate cost of capital for the option flows cannot be found.

But is it not illogical to assume that for each investment project the appropriate cost of capital can be found except for the flows associated with an option ? In the single period case for example, a (market)-valued twin-security or combination of (market)-valued securities is assumed to exist, permitting the reproduction of a project's flows so that its cost of capital can be established. Why would that be the case for projects except for sets of option flows. In the example at hand, a market-valued security with flows proportionate to 180 and 60 was found, permitting to derive a cost of capital of 20 %. Why then would a set of market-valued securities with payoffs proportional to -112 and -60 (or 67.68 and 0) not be found ?

Presenting future cash flows in a sweeping generalised manner (e.g. assuming normal distributions) permits the derivation of general truths. However, at the specific project level :

- it is mostly inaccurate (e.g. normality) ;

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6 It does seem odd however that few complain much about the widespread (incorrect) use of an across projects constant cost of capital but emit dire warnings against the error of applying the same (incorrect) method in evaluating option flows. Yet, the error in the first case is of the same nature and the same order of magnitude as the one in the latter case.'

7 The option method incorporates implicitly the correct cost of capital because the option flows are expressed as a linear combination of flows whose cost of capital is supposedly correctly known.

- it does not exploit the information usually available in the firm, particularly the information of a conditional nature ;
- usually a specific time-path for the results is not presented ;
- it does not allow much for active communication and discussion among those responsible nor it is easily subject to (audit)-reviews.

Therefore, rather than relying on the indirect way to estimate the cost of capital (via twin-security), we prefer the direct determination, project by project thru the acceptance of a 'best' correlation or covariance measure.

In the example at hand, determination of the correlation measure (R) is easy. Indeed, the project flows as well as the option flows are uniquely associated with 2 states of nature each with p = .5. So R = 1.0 in both cases. The distribution of market return is external information and an estimate easily available. In the example used, the implied market return has the following characteristics :

$r_{m1} = + .32$

$r_{m2} = + -.08$

$r_{\bar{m}} = + .12$

$\sigma^2_m = + .04$

The potential payoffs of the project are 180 and 60. If $V_j$ indicates the cash flow, $\sigma_v = 60$. Therefore,

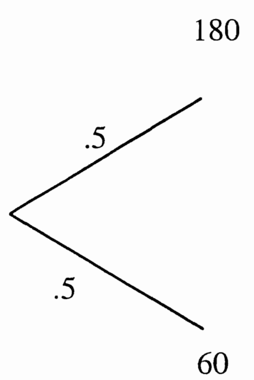
Cov $(V,r_m) = 1.(.20)(60) = 12$ (original situation)

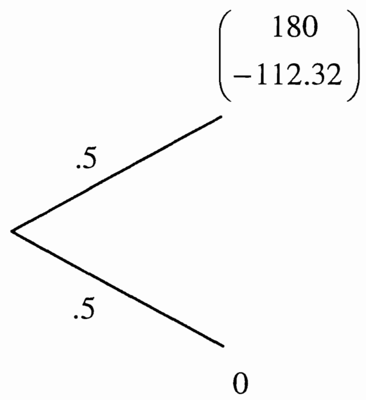
Cov $(V,r_m) = 1.(.20)(33.84) = 6.768$ (under waiting alternative)

We have summarized all NPV valuations on the next page[8].

*Project*

| Original situation | Situation under waiting alternative |
|---|---|



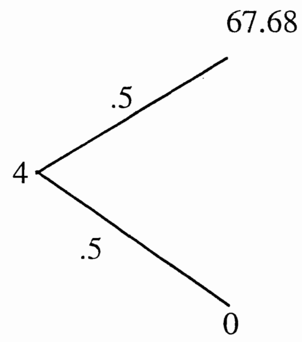



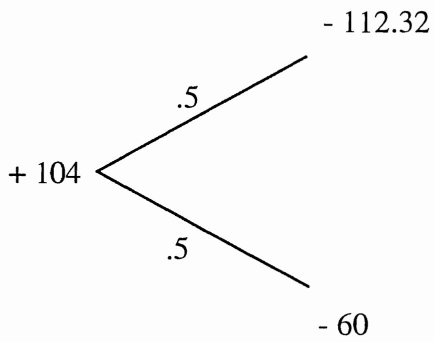

$$P = \left[120 - \frac{.04\ (12)}{.04}\right] \ /\ 1.08 - 104$$
$= -4$

$$P = \left[33.84 - \frac{.04\ (6.7568)}{.04}\right] \ /\ 1.08$$
$= 25.1$

*Option*

| Option value (net results) | Option value (changes cash flows) |
|---|---|

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8 On the basis of

$$P_j = \frac{V_j}{1 + E(r_j)} = \frac{V_j}{1 + (r_f + \frac{E(r_m) - r_f}{\sigma^2_{r_m}} \ .\ \mathrm{Cov}\ r_j \quad r_m\ )}$$

or after rearranging :

$$P_j = \left[V_j - \frac{E(r_m) - r_f}{\sigma^2_{r_m}} \ .\ \mathrm{Cov}\ (V_j, r_m)\right] \ /\ 1 + r_f$$

$$P = \left[33.84 - \frac{.04\ (6.768)}{.04}\right] /\ 1.08 \qquad P = \left[\frac{-86{,}16 + \frac{0.4}{0.4}(5.232)}{1.08}\right] + 104$$

$$= 29.1 \qquad\qquad = 29.10^{*}$$

* R is here = -1 (best results market, worst results option and vice-versa).

## 3. A multiperiod context

Assume a project with a duration of three years that does not meet the hurdle rate of 20%. However, carrying out the project delivers the promise of participation three years hence in a potentially lucrative market. Suppose the investment three years hence (project II) would be 900 and the present value of the future cash flows of project II at that time equal to 800.

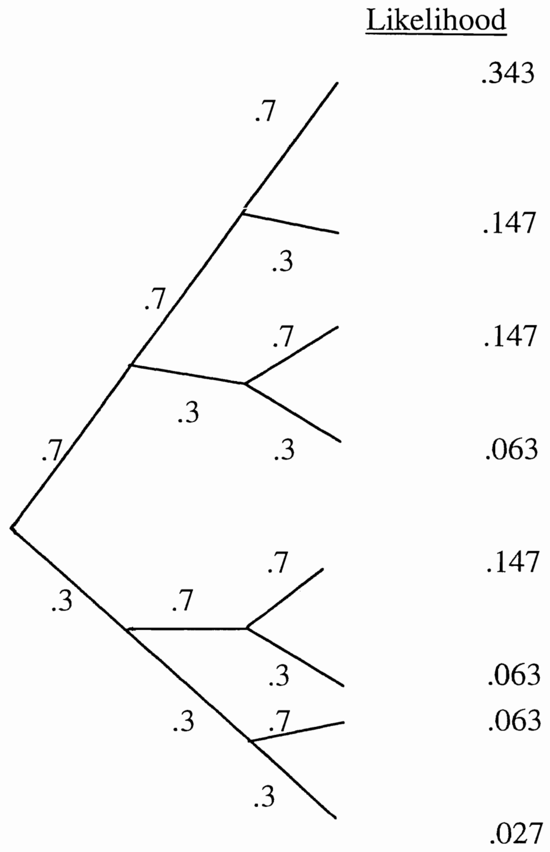
A present value of those cash-flows would be $800/(1.20)^3$ (where $\frac{800}{(1.20)^3}$ then serves as the value at present of the asset) and assuming a standard deviation of $.35\sqrt{T}$ or $.35\sqrt{3}$ and an exercise price of 900 (investment cost) the application of Black-Scholes expression puts the value of this option at 55."[9].

There are a number of difficulties with this solution. First, discounting the flows of a project succeeding a prior project at the prior's project cost of capital is in principle incorrect. For this procedure to be valid, flow patterns of prior and successor projects should show the same association with market returns. This is generally not the case, particularly not when the successor project is supposed to capture the benefits (+) from the sacrifices (-) made on its behalf in the prior project.

To explore this example a little bit further, suppose that market returns are N(0.12 ; 0.20). Project II - cash flows are N(800 ; 485). Discounting at 20 % implies that the two distributions are not independent. In fact, at 20 %, the one-period certainty equivalent of 800 is equal to 720 (given a risk free rate of 8 %) from which a covariance of 80 and a correlation coefficient between project II cash flows and market returns of 0.827 is derived. As stated, the procedure employed assumes that cash flows of project I (negative NPV) and of project II (hopefully positive NPV) bear the same relation to market returns ; in this case a correlation of 0.827.

Using a general distribution gleaned from the past for a specific future project can be labelled the extrapolation approach as against the analytic approach where in the latter case a careful examination of all relevant aspects is made to arrive at a correlation measure. In the practical firm context, such information is most clearly and easily presented in a bi-to multinominal context and it lends itself well to an analysis using the NPV-methodology.

Suppose that the probability of above average macro-economic growth in any given period over the next three years equals 70 %. Below average growth has a likelihood of 30 %.




The success of project II in our example could be tied to macro-economic growth (example : cyclical sector) or could e.g. be more dependent upon the development of human capital and research capability over the coming three years (pharmaceutical,

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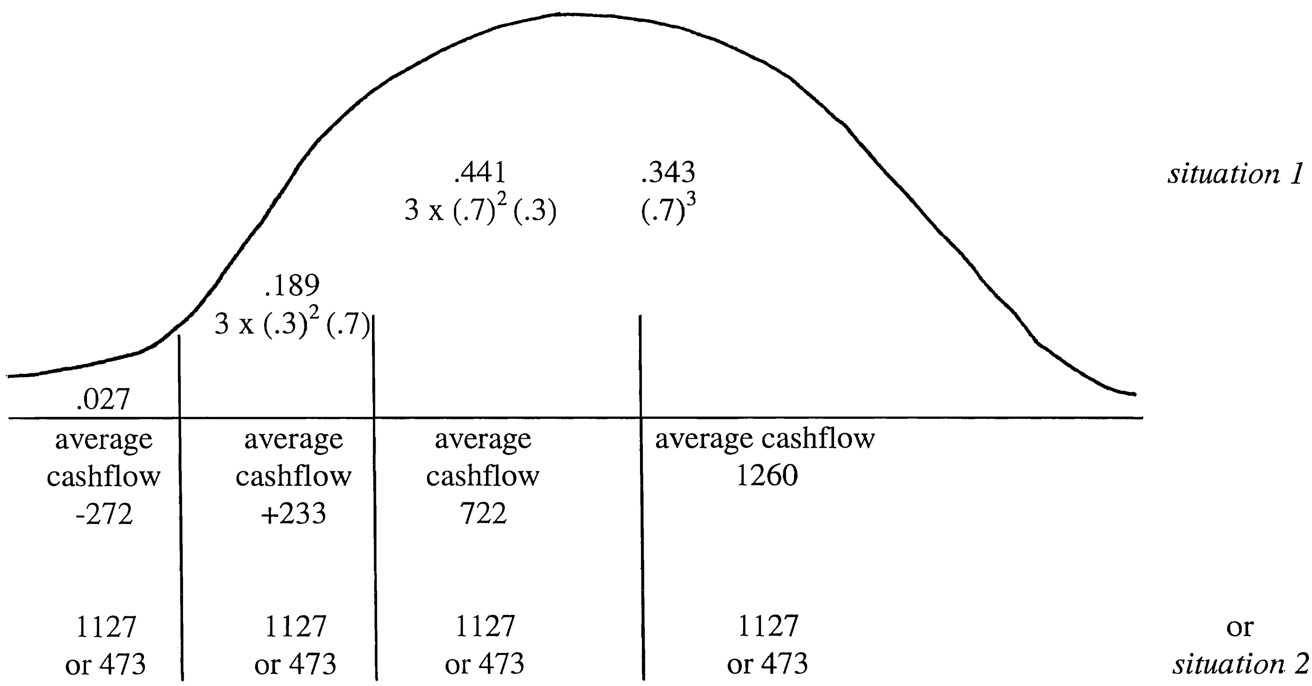
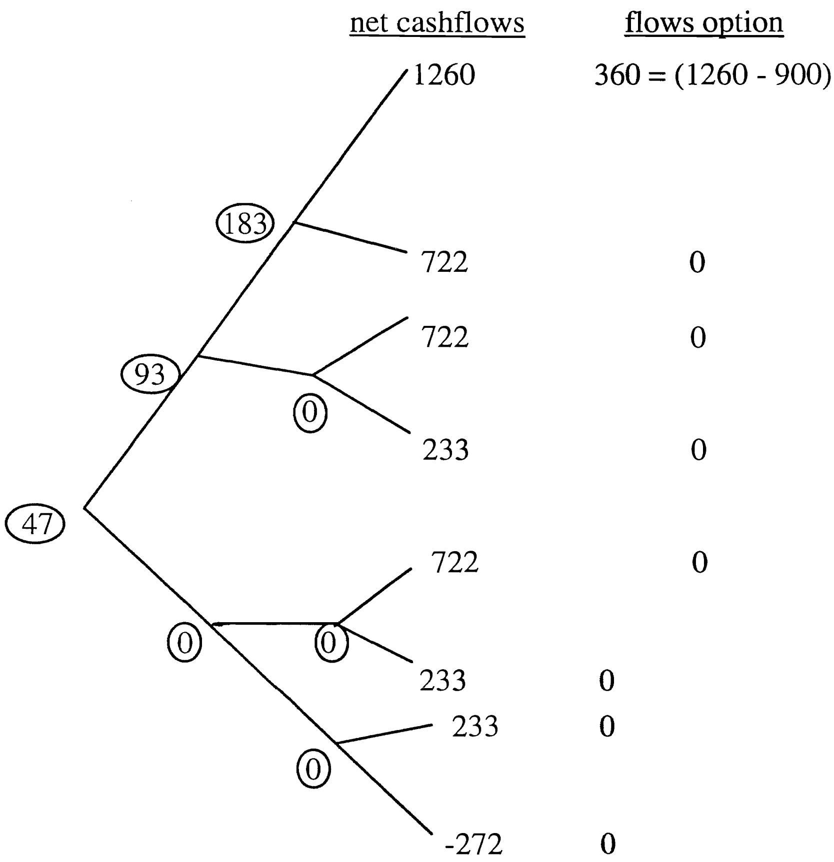
9 Brealy and Myers "Corporate Finance", third ed., MacGraw-Hill, p. 499.

telecommunication industries). These different situations might be pictured as follows (from a N : 800, 485).




where in situation 2 the average of the upper half and lower half of the given distribution is used.

If market returns (with a given constant spread) follow the macro-economic development, then in the first case project II returns and market returns are perfectly correlated (correlation coefficient equals one). Suppose market returns are either 20 % or -6.5 % (mean 12%, $\sigma = .12$) and the risk free rate 8 %, then the value of project II (as option to I) equals 47 as is calculated below.




The values 183, 93 and ultimately 47 have been obtained using the valuation expression reproduced as footnote 8.

$$\text{E;g. } 183 = \frac{\left[(.7)(360)+(.3)(0)\right] - \dfrac{.04}{.0144}\left[(108)(.08)(.7)+(-252)\cdot(-.185)(.3)\right]}{1.08}$$

Because market returns and project II returns (option to I) are uncorrelated in the second case, the value of the option (project II) is now :

$$\frac{\left[(.7)(360)+(.3)(0)\right]}{(1.08)^3} = 200$$

To summarize, market returns and variances over long periods are available, can be estimated and forecasted. Real project flows however are always in the future, have no history, are unique and may present specific associations. Therefore, sweeping extrapolations fed in options valuing formulae will result in crude guesses which can considerably be improved upon using specific information available. The N.P.V. methodology can correctly handle such information.

## 4. An operational algorithm

We assume that the available information about the future potential states of nature with respect to market returns are summarized as a probability tree. The number of periods and states of nature per period only depend on the useful information available and are not restricted in any operational sense. The number of states of nature per period equals the number of potentially different market returns.

Such a probability tree represents the marginal distribution of market returns over time. To be able to impose the correct risk premium on any particular project's cash flow, management must indicate, given the forecasted market returns, the project payoffs associated with the market returns (covariance function).

Given this probability tree setup (and per period),

Let y indicate the scaled payoff for a given state of nature, whereby :

y = 1.0 with probability p

y = 0.0 with probability (1 - p) and

p = probability of a given state of nature occurring in a given period.

Then E (y) = p

$$PV\ (y) = \left[p - \lambda\ \ \text{Cov. } y, r_m\right] / 1 + r_f$$

$$= p - \lambda\ \left[1\,(r_m - r_{\bar{m}})\,.\,p + 0\,.\,(r_m - r_{\bar{m}}\ (1 - p)\right]$$

$$= p\ \left[1 - \lambda\ \ (r_m - r_{\bar{m}})\right] / 1 + r_f$$

where $\lambda$ represents the relative price of risk, $r_{\bar{m}}$ the average market return and $r_m$ the market return for a given state of nature.

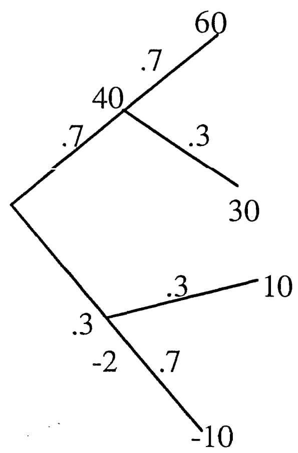
Given k potential states of nature per period, the discounted value of $\sum_k yk$ equals (over one period) :

$$\sum_{i=1}^{k} \frac{p_i \left[1-\lambda_i \left(r_{m_i} - r_{\bar{m}}\right)\right]}{1+rf_i}$$

Remark that $\lambda_i$, $r_{\bar{m}i}$ and $r_{fi}$ have the same value for all states of nature emanating from a given state of nature in the previous period.

Example

Suppose the relevant return distributions for an investment project can be represented as:




together with:

| | | | |
|---|---|---|---|
| $r_{11}$ | = | 0.18 [10] | |
| $r_{12}$ | = | 0.02 | $\bar{r}_1 = 0.10$ |
| $r_{21}$ | = | 0.18 | |
| $r_{22}$ | = | 0.02 | $\bar{r}_{21} = 0.10$ |
| $r_{23}$ | = | 0.08 | |
| $r_{24}$ | = | - 0.03 | $\bar{r}_{22} = 0.05$ |
| $\sigma_m^2$ | = | 0.04 | |
| $rf_1$ | = | 0.06 | |
| $rf_{21}$ | = | 0.06 | |
| $rf_{22}$ | = | 0.03 | |

[10] 0.18 and 0.02 are the expected returns of respectively the upper half and lower half of $\int z f(z)$ where $f(z)$: N (0.10; 0.20).

The discounted value amounts per unit potential pay off at each node are:

1.1: $[(0.7) . (1\text{-}1\ (0.18 - 0.10)]/1.06 = 0.608$

1.2: $[(0.3) . (1\text{-}1\ (0.02 - 0.10)]/1.06 = 0.306$

2.1: $[(0.7) . (1\text{-}1\ (0.18 - 0.10)]/1.06 = 0.608$

2.2: $[(0.3) . (1\text{-}1\ (0.02 - 0.10)]/1.06 = 0.306$

2.3: $[(0.3) . (1\text{-}0.25\ (0.08 - 0.05)]/1.03 = 0.289$

2.4: $[(0.7) . (1\text{-}0.25\ (-0.03 - 0.05)]/1.03 = 0.693$

Thus the estimated present value of the proposed project is:

(60 x 0.608 x 0.608) + (30 x 0.306 x 0.608) + (10 x 0.289 x 0.306)

+ (-10 x 0.693 x 0.306) + (40 x 0.608) + (-2 x 0.306) =

22.18 + 5.58 + 0.88 - 2.12 + 24.32 - 0.61 = <u>50.23</u>

The possibility for the firm to stop the project were the first period to show an unfavourable development would be worth (2.3 : $\Delta = -10$ ; 2.4 : $\Delta = +10$) :

-10 x 0.289 x 0.306

+10 x 0.693 x 0.306 or -0.88 + 2.12 = <u>+1.24</u>

The above can be generalized. Let K be the number of states of nature per period. The total number of states per period then is $K^J$ where $j = 1,2, \ldots N$. With the $a_jk$ being the discounted values per unit potential payoff, the present value of a project equals:

$$\sum_{j=1}^{N} \sum_{k=1}^{K^j} a_jk \ \ \beta_jk$$

where the $\beta_jk$ are the cash flows at the respective points. Options induce changes in the original cash flows at certain nodes whereas the $a_jk$ values are not affected. The new present value equals:

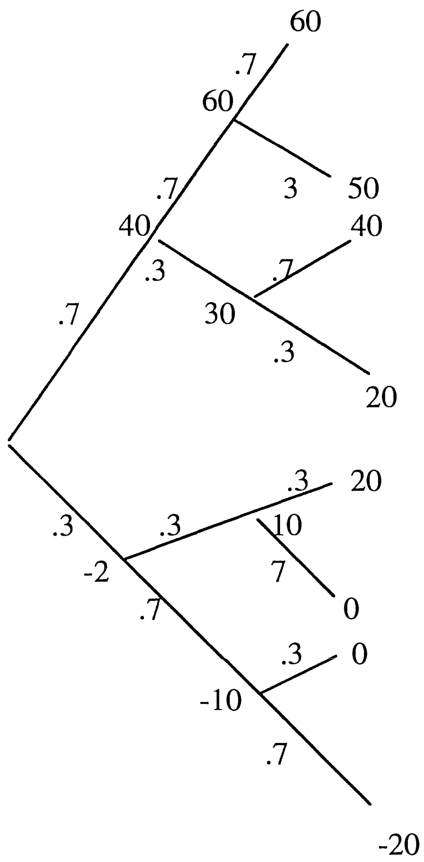
$$\sum_{j=1}^{N} \sum_{k=1}^{K^j} a_jk \ \ (\beta_jk + \Delta\, \beta_jk) \qquad \text{or}$$

$$\sum_{j=1}^{N} \sum_{k=1}^{K^j} a_jk \ \ \beta_jk \ \text{(value original project) plus}$$

$$\sum_{j=1}^{N} \sum_{k=1}^{K^j} = a_jk \ \ \Delta\, \beta_jk \ \text{(value option)}$$

The option value can be calculated using the existing $a_jk$.

As an illustration, we use the three-period case presented below.




together with

$r_{11} = 0.18$ $\quad \bar{r}_1 = 0.10$

$r_{12} = 0.02$

$r_{21} = 0.18$ $\quad \bar{r}_{21} = 0.10$

$r_{22} = 0.02$

$r_{23} = 0.08$ $\quad \bar{r}_{22} = 0.05$

$r_{24} = -0.03$

$r_{31} = 0.18$ $\quad \bar{r}_{31} = 0.10$

$r_{32} = 0.02$

$r_{33} = 0.18$ $\quad \bar{r}_{32} = 0.10$

$r_{34} = 0.02$

$r_{35} = 0.08$ $\quad \bar{r}_{33} = 0.05$

$r_{36} = -0.03$

$r_{37} = 0.08$ $\quad \bar{r}_{34} = 0.05$

$r_{38} = -0.03$

$r_{f1} = 0.06$ ; $r_{f21} = 0.06$ ; $r_{f22} = 0.03$ ; $r_{f31} = 0.06$ ; $r_{f32} = 0.06$ ; $r_{f33} = 0.03$ ; $r_{f34} = 0.03$ ;
$\sigma^2_m = 0.04$

The top part of the tree (flows $b_{11}$, $b_{21}$, $b_{22}$, $b_{31}$,$b_{32}$,$b_{33}$,$b_{34}$) represents flows in case of a favourable macro-economic development and vice-versa for the bottom part (flows $b_{12}$, $b_{23}$, $b_{24}$, $b_{35}$, $b_{36}$, $b_{37}$, $b_{38}$). An option is available to the extent that in case of an unfavourable development, the project can be stopped. Carrying out the project will also result in the acquisition of skills and market exposure permitting the establishment of a second follow-up project. Management estimates the value of the flows at three periods from now for this second project to be around 200 with an upper half average of 240 and a lower half average of 160. These results are estimated to be independent of the results of the first project. The cost for the first project is 75 and 150 for the second. If the first project is halted before completion, extra efforts will have to be made to maintain the prospects for project II. These efforts will cost 20 (18.85 in present value terms) for stopping after 1 period and 15 (14.15 in present value term) for stopping after 2 periods).

The $a_j k$ values for the first project are (as calculated from the data) :

| | |
|---|---|
| $a_{11}$ : 0.608 | $a_{31}$ : 0.608 |
| $a_{12}$ : 0.306 | $a_{32}$ : 0.306 |
| $a_{21}$ : 0.608 | $a_{33}$ : 0.608 |
| $a_{22}$ : 0.306 | $a_{34}$ : 0.306 |
| $a_{23}$ : 0.289 | $a_{35}$ : 0.289 |
| $a_{24}$ : 0.693 | $a_{36}$ : 0.693 |
| | $a_{37}$ : 0.289 |
| | $a_{38}$ : 0.693 |

The value of the first project is 73.70 against an investment cost of 75. The present value of the second project is 169. This value has been obtained as follows :

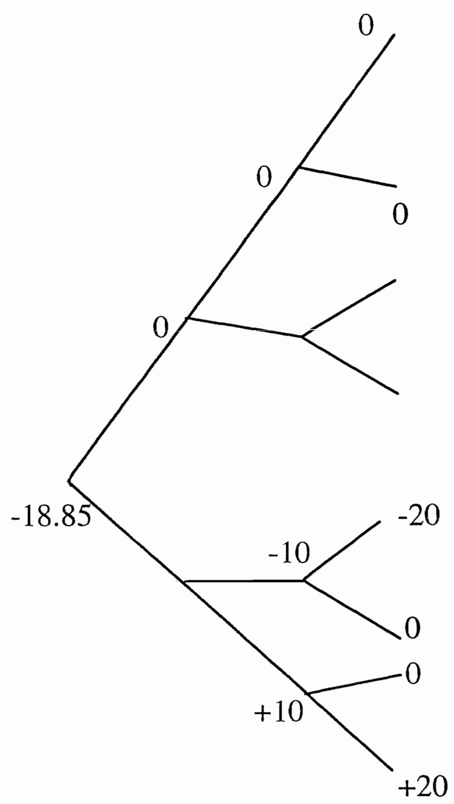
200 x .608 x .608 x .608 +
200 x .306 x.608 x .608 +
200 x .608 x .306 x .608 +
200 x .306 x .306 x .306 +
200 x .289 x .289 x .306 +
200 x .693 x .693 x .306 +
200 x .289 x .693 x .306 +
200 x .693 x .693 x .306

The value of the combined cash flows 37 and 38 of project I is negative [0, - 2.94 (-20 x .693 x .693 x .306)]. The same is true for the combined cash flows 23 (+ 35 and 36) and 24(+ 37 and 38), according to the calculations :

| | |
|---|---|
| 35 : 20 x .289 x .289 x .306 | = + 0.51 |
| 36 : 0 x | = 0.00 |
| 23 : 10 x .289 x .306 = | + 0.88 |
| 37: 0 x | = 0.00 |
| 38 : -20 x .693 x .693 x .306 = | -2.94 |
| 24 : -10 x .693 x .306 | = - 2.12 |
| | -------- |
| | -3.67 |

So, if project I stood by itself, its value would be 73.7 + 3.67 or 77.37. This value implies halting the project after one period in case of an unfavourable development.

However, project I and II are linked (II dependent upon I) so that a judgment should involve the joint project. The value of the joint project is (73.7 - 75) +(169 - 150) = 17.7. Halting the project after 1 year brings now the following changes in the combined flows :




| | | |
|---|---|---|
| value option : | + 3.67 | (see above) |
| extra effort for II | - 18.85 | |
| | ---- | |
| | = -15.18 | |

The value of the combined project reduces to 17.7 - 15.2 = 2.5. Likewise, the value of the joint project would diminish if the first project were halted after two years.

A change of the cash flows anywhere in the system can be easily and consistently evaluated with this present value framework. There is no limitation on the number of periods or number of states of nature per period. Programmation of models with an increased number of periods and/or states should insure quick calculation.

**Conclusion**

'Opportunities' associated with real investment projects imply changes in the 'original' flow pattern of the project. These changes can be evaluated either by making use of the option methodology or by using the traditional discounted cash flow methodology. Of course, the latter, as the former, should be correctly applied.

Real investment projects are quite specific future undertakings. The cash flows changes implied by 'flexibilities' accompanying them may exhibit patterns that do not fit well the distributions underlying traditional option valuation formulae. It seems to us that a complex structure of changes can be evaluated in a more correct way and with more insight using probability tree and NPV methodology.

Of course it remains so that the recognition of the 'opportunities' associated with a project deserves most attention and that errors of measurement in their valuation are of a lesser importance.

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